are replaced with equivalent uniform thin layers for the purposes of evaluation. These equivalent layers are more of an average electromagnetic thickness rather than a velocity boundary layer as Eq. (1) is particularly sensitive to the plasma frequency ω_p, which is also an average value for the layer.

Based upon the equivalent antenna-plasma model, the calculations show the windward-leeward attenuation variations can be explained in terms of an approximately equal electron density (and plasma frequency, ω_p) on both sides of the cylinder. The leeward side d is roughly four times that on the windward side, as shown in Table 1. The effective thickening is due to the complex separated vortex pattern rolling over from the windward side.

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Reinforced Composite Materials with Curved Fibers

Farhad Tabaddor* and C. H. Chen†
The B. F. Goodrich Co., Akron, Ohio

Introduction

The initial fiber misalignment in a unidirectional reinforced composite can affect, considerably, the mechanical properties in comparison to an ideal reinforced composite with fibers, being assumed straight. The subject has been considered by a number of investigators. Among them, Tarnopolsky et al.,[1] used a mechanics of material approach to obtain the axial Young's modulus of a fiber-reinforced composite ply with fiber misalignment only in the plane of the ply. They assumed that the fiber misalignment is uniform and symmetric with respect to the longitudinal axis, so that the fiber path can be represented reasonably well by a sinusoidal curve (Fig. 1a). They then considered a segment of length dx (Fig. 1b) and obtained the Young's modulus in x direction of this segment in terms of the properties of straight-fiber composite by using transformation formula. This Young's modulus of the segment was then integrated over a length of a period to yield a closed form expression for the apparent Young's modulus in x direction. It is reported in Ref. 1 that good agreement was obtained between theory and experiment. Other experimental verification on the effect of curved fibers can be found in Ref 2.

In this Note, using the result for Young's modulus obtained in Ref. 1, the one-dimensional stress-strain relation of a composite with curved fiber will be derived, and the effect of misalignment on this stress-strain relation will be studied.

Basic Relations

We will consider the two-dimensional problem of the unidirectional reinforced composite with curved fibers subjected to unidirectional loading. The fiber, as shown in Fig. 1a, is

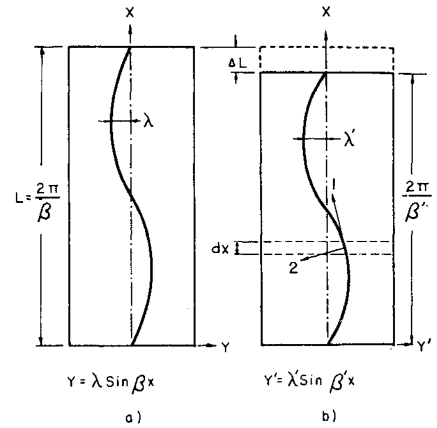

Fig. 1 Basic element a) before and b) after deformation.

assumed to have sinusoidal misalignment, expressed by

$$y = \lambda \sin\beta x \tag{1}$$

where y measures the deviation of a typical fiber from the average position, and λ and $\beta = 2\pi/L$ are amplitude and frequency of misalignment, respectively. Figure 1b shows the same fiber after deformation as a result of loading in x direction. Upon the application of the load, the material deforms, and the reinforcing fiber would assume a new sinusoidal form given by

$$y = \lambda' \sin\beta' x \tag{2}$$

The Young's modulus in x direction of a composite with fibers having the path of Eq. (2) is given, according to Ref. 1, as

$$\frac{1}{E_x} = \frac{1}{2}\left(\frac{1}{E_{11}} - \frac{1}{G_{12}} + \frac{2\nu_{12}}{E_{11}} + \frac{1}{E_{22}}\right)[2 + (\lambda'\beta')^2] \times [1 + (\lambda'\beta')^2]^{-3/2} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} - \frac{2}{E_{22}}\right) \times [1 + (\lambda'\beta')^2]^{-1/2} + \frac{1}{E_{22}} \tag{3}$$

where E_{11}, E_{22}, G_{12}, and ν_{12} are the axial Young's modulus, transverse Young's modulus, longitudinal shear modulus, and Poisson's ratio, respectively, of unidirectional composite with straight fibers. These quantities can be evaluated by various micromechanical theories if the properties of constituent materials and the fiber volume ratio (v) are known. If in addition the quantity $\lambda'\beta'$ can be related to the given initial conditions of λ, β and to the amount of strain ϵ, then E_x can be evaluated from Eq. (3). Thus we have relation of E_x vs ϵ, which usually is a nonlinear in nature. Knowing E_x corresponding to a particular value of ϵ, the stress required to produce that strain is given in conventional manner as

$$\sigma_x = E_x \epsilon \tag{4}$$

which gives us, in general, nonlinear stress-strain relation of a composite under our consideration.

Let us observe from Eq. (2) that the amplitude and frequency of misalignment appear as a product and therefore

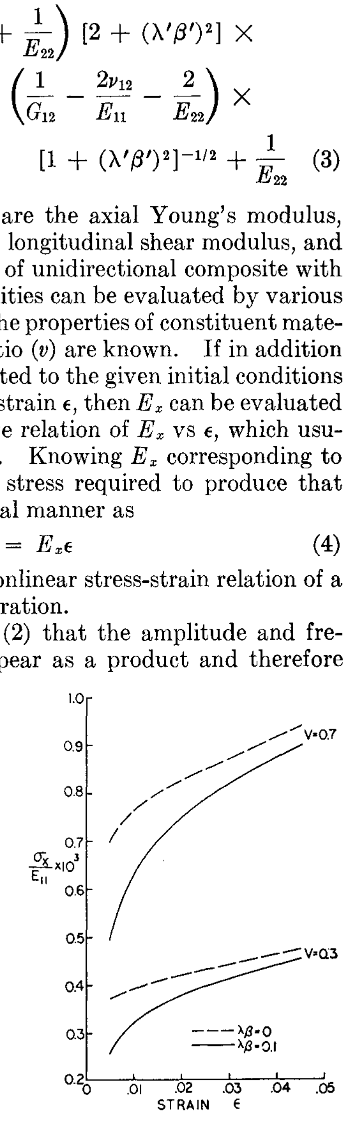

Fig. 2 Nondimensional stress-strain relations for reinforced composites with ν_m = 0.49, ν_f = 0.33, and G_f/G_m = 3000.

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* Senior Engineering Scientist.
† Senior Engineering Scientist. Member AIAA.

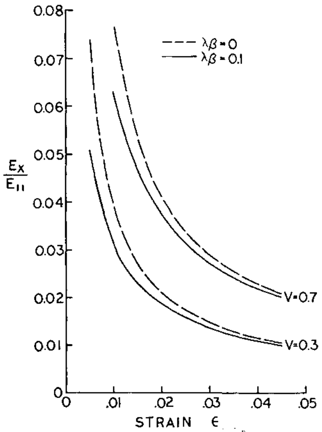

Fig. 3 Nondimensional Young's modulus vs strain for reinforced composites with $\nu_m = 0.49$, $\nu_f = 0.33$, and $G_f/G_m = 3000$.

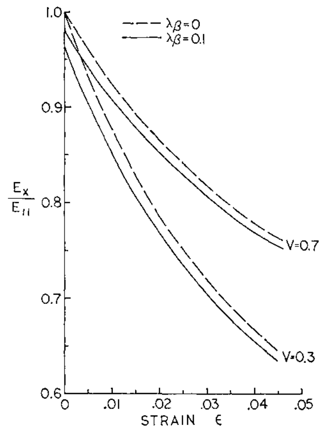

Fig. 5 Nondimensional Young's modules vs strain for reinforced composites with $\nu_m = 0.49$, $\nu_f = 0.33$, and $G_f/G_m = 20$.

should be treated as one single parameter. The physical implication is that the class of all sinusoidal misalignments associated with the same $\lambda\beta$ would have the same effect on Young's modulus. Therefore, this nondimensional parameter characterizes the degree of misalignment.

The key point to the aforementioned procedure is the evaluation of E_x in terms of λ, β, and ϵ. In what follows, we will present a simple, yet reasonable, way of obtaining this relation from purely geometrical consideration. Referring to Figs. 1a and 1b we have

$$2\pi/\beta' = (2\pi/\beta)(1 - \epsilon) \tag{5}$$

where $\epsilon = \Delta L/L$. This gives us

$$1/\beta' = (1 - \epsilon)/\beta \tag{6}$$

Here the compression strain is taken as positive strain. We will assume that the length of the reinforcing elements remain unchanged during the deformation as long as the fiber remained curved. This implies that the strains in reinforcing fibers are negligible compared to those of geometry change. As a consequence, we have the following relation:

$$\int_0^{2\pi/\beta} ds = \int^{2\pi/\beta'} ds' \tag{7}$$

where ds and ds' are infinitesimal arc lengths of a typical fiber before and after loading, respectively, and can be expressed as

$$\begin{aligned} ds &= [1 + (\lambda\beta \cos\beta x)^2]^{1/2}dx \\ ds' &= [1 + (\lambda'\beta' \cos\beta' x)^2]^{1/2}dx \end{aligned} \tag{8}$$

Integration of Eq. (7), with the help from Eq. (8), yields

$$\frac{[1 + (\lambda\beta)^2]^{1/2}}{\beta}\left[1 - \left(\frac{1}{2}\right)^2 K^2 - \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 K^4 - \ldots\right] = \frac{[1 + (\lambda'\beta')^2]^{1/2}}{\beta'}\left[1 - \left(\frac{1}{2}\right)^2 K'^2 - \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 K'^4 - \ldots\right] \tag{9}$$

where $K^2 = (\lambda\beta)^2/[1 + (\lambda\beta)^2]$ and $K'^2 = (\lambda'\beta')^2/[1 + (\lambda'\beta')^2]$. Equation (9) together with Eq. (6) gives us an implicit relation between $\lambda'\beta'$ and $\lambda\beta$. For problems of practical interest we can safely restrict ourselves to the case where

$$\lambda\beta \ll 1,\ \lambda'\beta' \ll 1 \tag{10}$$

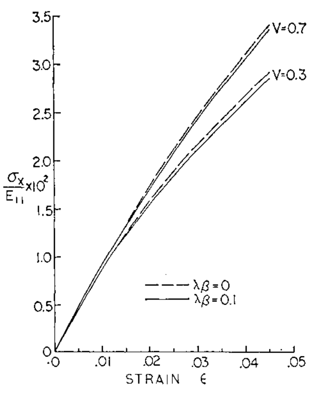

Fig. 4 Nondimensional stress-strain relations for reinforced composites with $\nu_m = 0.49$, $\nu_f = 0.33$, and $G_f/G_m = 20$.

If Eq. (9) is expanded and terms of higher than second order are neglected, then by making use of Eq. (6) we finally obtain

$$\lambda'\beta' = 2\{[1 + (\lambda\beta/2)^2]/(1 - \epsilon) - 1\} \tag{11}$$

Substitution of Eq. (11) into Eq. (3), and the latter into Eq. (4), gives us an approximate explicit stress-strain relation.

From Eq. (11) we can see that when ϵ is positive (compression), the quantity $\lambda'\beta'$ is always positive. However when ϵ is negative (tension), the argument inside the bracket becomes positive or zero only when $|\epsilon| \leqslant (\lambda\beta/2)^2$. For $\lambda\beta = 0.1$, for example, we then have $|\epsilon| \leqslant 0.0025$. This means that the value of E_x approaches E_{11} very rapidly in the negative (tensile) strain range, despite the relatively large initial imperfection or misalignment. This is to be expected since the curved fibers tend to straighten out under tension.

Numerical Results

Figures 2 and 3 present the numerical results obtained from using Eqs. (11), (3), and (4). Figure 2 presents the normalized stress-strain curves, and Fig. 3 shows the normalized Young's modulus (E_x/E_{11}) vs strain. The notations G_m, G_f, ν_m, ν_f, and v denote matrix shear modulus, fiber shear modulus, matrix Poisson's ratio, fiber Poisson's ratio, and fiber volume content, respectively. The quantities E_{11}, E_{22}, G_{12}, and ν_{12} which are needed in Eq. (3) are calculated by using an approximate best closed-form solution as given in Ref. 3. Although it is realized that these equations are not the best among various micro-mechanical solutions available, it is felt that they are adequate for illustration purpose. Figures 2 and 3 are for a composite with a matrix very soft relative to the fibers ($G_f/G_m = 3000$), and Figs. 4 and 5 are for one with a stiffer matrix ($G_f/G_m = 20$). Figures 2 and 4 clearly indicate the nonlinearity of the stress-strain relation, even for small strain range. This nonlinearity is more pronounced for the composite with the softer matrix. The differences between the values of E_x/E_{11} and unity at zero strain on Figs. 3 and 5 indicate the reduction of stiffness due to initial curvature. As was stated earlier, this normalized modulus approaches unity very rapidly in the negative strain range (tension) for $\lambda\beta > 0$ (see Fig. 5).

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